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The General Relativistic Hydrogen Atom

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Abstract. The general relativistic Dirac equation is formulated in an arbitrary curved space-time using differential forms. These equations are applied to spherically symmetric systems with arbitrary charge and mass. For the case of a black hole (with event horizon) it is shown that the Dirac Hamiltonian is self-adjoint, has essential spectrum the whole real line and no bound states. Although rigorous results are obtained only for a spherically symmetric system, it is argued that, in the presence of any event horizon there will be no bound states. The case of a naked singularity is investigated with the results that the Dirac Hamiltonian is not self-adjoint. The self-adjoint extensions preserving angular momentum are studied and their spectrum is found to consist of an essential spectrum corresponding to that of a free electron plus eigenvalues in the gap $(-mc^2, +mc^2)$. It is shown that, for certain boundary conditions, neutrino bound states exist.

Introduction

In this paper we formulate the Dirac equation in an arbitrary curved space-time (Sect. I). These equations are applied to spherically symmetric systems with arbitrary charge and mass (Sect. II and III). For the case of a black hole (with an event horizon) we show that the Dirac Hamiltonian is self-adjoint, has essential spectrum the whole real line and no bound states (no point spectrum) (Sect. IV). Although rigorous results are only obtained for the case of spherical symmetry, it is argued that in the presence of any event horizon there will be no bound states.

In Sect. V the case of a naked singularity is investigated. For a proton or any atom in the periodic table, the charge to mass ratio is such that the corresponding Riesner-Nordstrom solution has a naked singularity. For the case of a naked singularity we show the Dirac Hamiltonian is not self-adjoint. The deficiency indices are (∞, ∞) but for each angular momentum subspace $\mathfrak{M}_{km}(k = \pm 1, \pm 2, \pm 3, \ldots, m = \pm 1/2, \pm 3/2, \ldots)$ labelled by two quantum numbers k and m, the

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