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Geometric Analysis of ϕ^4 Fields and Ising Models. Parts I and II

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Abstract. We provide here the details of the proof, announced in [1], that in $d>4$ dimensions the (even) ϕ^4 Euclidean field theory, with a lattice cut-off, is inevitably free in the continuum limit (in the single phase regime). The analysis is nonperturbative, and is based on a representation of the field variables (or spins in Ising systems) as source/sink creation operators in a system of random currents – which may be viewed as the mediators of correlations. In this dual representation, the onset of long-range-order is attributed to percolation in an ensemble of sourceless currents, and the physical interaction in the ϕ^4 field – and other aspects of the critical behavior in Ising models – are directly related to the intersection properties of long current clusters. An insight into the criticality of the dimension $d=4$ is derived from an analogy (foreseen by K. Symanzik) with the intersection properties of paths of Brownian motion. Other results include the proof that in certain respect, the critical behavior in Ising models is in exact agreement with the mean-field approximation in high dimensions $d>4$, but not in the low dimension $d=2$ – for which we establish the "universality" of hyperscaling.

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