Commun. Math. Phys. 85, 449–470 (1982)

Communications in
Mathematical
Physics

Symmetric Random Walks in Random Environments

V. V. Anshelevich[1], K. M. Khanin[2], and Ya. G. Sinai[2]

1 Institute of Molecular Genetics, Academy of Science of USSR, SU-123182 Moscow, USSR
2 L. D. Landau Institute for Theoretical Physics, Academy of Science of USSR, SU-117334 Moscow, USSR

Abstract. We consider a random walk on the d-dimensional lattice $\mathbb{Z}^d$ where the transition probabilities $p(x,y)$ are symmetric, $p(x,y)=p(y,x)$, different from zero only if $y-x$ belongs to a finite symmetric set including the origin and are random. We prove the convergence of the finite-dimensional probability distributions of normalized random paths to the finite-dimensional probability distributions of a Wiener process and find our an explicit expression for the diffusion matrix.

1. Formulation of the Problem and Results

We shall consider Markov chains whose phase space is the cubic d-dimensional lattice $\mathbb{Z}^d$. In the case of discrete time such chains are defined by their transition probabilities $p(x,y)$, $x\in\mathbb{Z}^d$, $y\in\mathbb{Z}^d$ which are replaced by differential transition probabilities $w(x,y)$, $x\in\mathbb{Z}^d$, $y\in\mathbb{Z}^d$ in the case of continuous time. We shall discuss the situation when $p(x,y)$ or $w(x,y)$ are random variables not depending on time. One says in these cases that one has a random walk in a random environment (see [1–2]).

There are many physical problems where one encounters similar random walks. We can mention some problems in crystallography (see [3]), and biophysics [4]. In this spirit one can discuss kinetic properties of Lorentz gas with random configurations of scatterers.

The one-dimensional case with possible transitions $x\to x\pm 1$ is mostly investigated from the mathematical point of view. The first results are due to Kesten, M. Kozlov, and Spitzer (see [1]). One can also mention the papers [5–6]. In [6] the case when $p(x,x+1)$ and $p(x,x-1)=1-p(x,x+1)$ are identically distributed was considered. An unexpected result of [6] is that the random walk can be highly nonuniform and a moving point spends an unusually large part of time in some regions of $\mathbb{Z}^1$. The positions of these regions and the distribution of time depend on a realization of probabilities $p(x,x+1)$.

Quite a different situation arises if one admits the transitions $x\to x-1, x, x+1$ and adds the symmetry condition $p(x,y)=p(y,x)$ or $w(x,y)=w(y,x)$. This case is

0010-3616/82/0085/0449/$04.40