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Borel Summability for a Nonpolynomial Potential

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Abstract. We consider the energy levels of a one-dimensional quantum system in the rational potential $\frac{1}{2}x^2 \pm gx^4/(1+\alpha g x^2)$. Their perturbation expansions in g are shown to be Borel summable. The proof is flexible enough to allow simple extensions to other nonpolynomial interactions.

I. Introduction

In recent years much attention has been devoted to the question of the Borel summability of perturbation expansions for quantities of interest in various quantum mechanical systems and quantum field models. Proofs of such a summability property have been given, e.g. for the energy levels of anharmonic oscillators [1] and for field-theoretic analogues, namely the Schwinger functions of super-renormalizable models (φ_2^4 and φ_3^4 theories) [2]. The purpose of this paper is to investigate the Borel summability of the perturbation expansion of the energy levels in a one-dimensional model with a "singular" (nonpolynomial) potential. Specifically, we shall consider the Hamiltonian

$$H^{\pm}(g) = \tfrac{1}{2}p^2 + V^{\pm}(x;g) \equiv \tfrac{1}{2}p^2 + \tfrac{1}{2}x^2 \pm \frac{gx^4}{1+\alpha g x^2} \tag{I.1}$$

on the Hilbert space $L^2(-\infty,\infty)$, where the "physical" range of the parameters g and α is

$$\left.\begin{array}{l} g \geq 0\,, \\ \alpha > 0 \text{ in the "+" case}\,, \\ \alpha > 2 \text{ in the "−" case [in order that } V^-(x;g) \to +\infty \text{ for } x^2 \to \infty]\,. \end{array}\right\} \tag{I.2}$$

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