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On Generalizations of the KMS-Boundary Condition

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Abstract. We investigate the possibility to generalize the KMS-boundary condition for a thermodynamical system by following essentially the same procedure that for a finite system would amount to choosing a certain class of more general density functions on phase space (or density matrices) than the ones corresponding to the canonical or grand-canonical ensemble.

1. Introduction

Gibbs states of thermodynamical systems in classical physics are described by means of conditional probabilities that are related to the energy of interaction of the configurations of finite systems with the outside world by means of an exponential function.

In [1] states of classical systems are considered where the above mentioned exponential function is replaced by a set of more general (albeit invertible) functions. These states are called regular conditional equilibrium (CE) states. A subclass of these states can, under suitable conditions, be shown to be Gibbs states [1]. The CE states are to be viewed as states obtained as thermodynamical limits of microcanonical states for finite systems. On the other hand Gibbs states are thermodynamical limits of canonical states for finite systems with specified boundary conditions (or convex combinations thereof) (cf. [2]).

Once one can show as in [1] that, under suitable conditions, regular CE states are Gibbs states, then one has made statements about the "equivalence of ensembles." Loosely speaking one can say that the exponential function, representing the Gibbs character of the state, is the thermodynamical significant object.

For quantum systems, under suitable conditions on the way one performs the thermodynamic limit on the finite volume correlation functions and the dynamics, one obtains the well-known KMS-boundary condition [3]. The KMS-boundary condition as introduced in [3] reads:

$$\int_{-\infty}^{\infty} \omega(A\alpha_t(B))\, f(t-i\beta)\, dt = \int_{-\infty}^{\infty} \omega(\alpha_t(B)A)\, f(t)\, dt \qquad (1)$$

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