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Local Extensions in Singular Space-Times II

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Abstract. Previous results of the author are corrected by reformulating them in space-times whose Riemann tensor satisfies a Hölder condition.

1. Introduction

In an earlier paper with this title [2] I showed the existence of local extensions through quasiregular singularities (in the terminology of [5]) by (implicitly) assuming that a spacetime with a C^{k-2} Riemann tensor had a C^k metric. This assumption may not be correct (the alleged proof which I gave in [3] being invalid). The basic results do hold, however, if one uses $C^{k,\alpha}$ conditions (a Hölder condition with exponent α, $0<\alpha<1$, on the k^{th} derivative). The technical tools needed to modify the proof are given in detail in [4]; the aim of the present paper is to outline their application to local extensions.

We first clarify the term "local extension" of a spacetime (M, g), of which there are two definitions in the literature. Here, and in [5], it means an isometry $\phi: U \to M'$, where $U \subset M$ and (M', g') is a spacetime, such that

(i) U contains a curve γ which is incomplete with respect to a generalised affine parameter and inextendible in M.

(ii) $\phi \circ \gamma$ is extendible in M'.

Hawking and Ellis [6], on the other hand, replace (i) by the condition that $\bar{U}$ is not compact in M, and (ii) by the condition that $\phi(U)$ is compact in M'. This has the undesirable consequence that Minkowski space is locally extendible [1]. With the author's definition, certain compact space-times having trapped geodesics may be locally extendible.

2. Results

The theorem will be formulated for the case where γ in the definition above is a broken geodesic. Since any rectifiable curve can be approximated by a broken geodesic this is no loss of generality, and it enables us to give a concrete description of the set U that can be extended. In addition we impose a restriction ((iv) below) that corresponds to the non-spiral condition imposed in the earlier paper [2]. The theorem will only be proved for $C^{0,\alpha}$ Riemann tensors; but it is clear that the procedure extends to $C^{k,\alpha}$.

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