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Compactness and the Maximal Gibbs State for Random Gibbs Fields on a Lattice

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Abstract. We prove for a general class of Gibbsian Random Field on $\mathbb{Z}^\nu$ that the set of tempered Gibbs states is compact. This class contains the Euclidean random fields. Moreover if the interaction is attractive, there is a unique minimal and maximal Gibbs state μ_- and μ_+. $\mu_\pm$ are unique translation invariant and have the global Markov property. We also prove that uniqueness of the tempered Gibbs state is equivalent to the magnetizations $m_\pm = \mu_\pm(q_x)$ being equal which is true if the pressure is differentiable.

Introduction

It is well known in statistical mechanics, that any statistical mechanical system with a compact state space has a compact set of Gibbs states. We prove in this paper, by utilizing a criterion that goes back to Dobrushin [11], that also for statistical mechanical systems with a non-compact state space the set of Gibbs states is compact provided the interaction satisfies certain conditions, and we consider as Gibbs states only the tempered Gibbs states. In fact we prove that the set of tempered Gibbs states form a Choquet simplex. This holds especially for the Euclidean lattice fields, and also for a much wider class of lattice interactions given by one and two-body forces.

The compactness of the Gibbs states gives us the existence of the maximal μ_+ and minimal μ_- Gibbs state in the case of an attractive interaction. Let us point out that the Euclidean lattice fields have attractive interaction. In the case of compact state space and attractive interactions μ_+ and μ_- were introduced by Preston [40] which also proved that they were pure. Later Fölmer [19] pointed out that they also have the global Markov property if the corresponding interaction is Markov. Using the compactness for the tempered Gibbs states in the case of non-compact fiber, we are able to prove not only that the maximal and minimal Gibbs states exists, but also that they are pure, translation invariant and have the global Markov property. Moreover the set of tempered Gibbs

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