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Analytic Interpolation and Borel Summability of the $(\frac{\lambda}{N}|\Phi_N|^{:4})_2$ Models

I. Finite Volume Approximation

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Abstract. Analytic interpolation in the variable $1/N$ of $\left(\frac{\lambda}{N}|\Phi_N|^{:4}\right)_2$ models is constructed at finite volume approximation. We prove Borel summability of the Taylor series at $1/N=0$ of their Schwinger functions. We also give an extension of the domain of analyticity in the coupling constant.

Introduction

We study an analytic interpolation and the asymptotic behaviour of a family of vector quantum fields, self-coupled with a quartic interaction, in a two dimensional space-time. So we carry on the study of the "$\frac{1}{N}$ expansion" for the family of $(\frac{\lambda}{N}\ |\Phi_N|^{:4})_2$ models, initiated by Kupiainen [2].

More precisely, for each integer N, we start with the Schwinger functions of a vector field Φ_N, with N components, submitted to the $\frac{\lambda}{N}|\Phi_N|^{:4}$ interaction; their (momentum and volume cut-off) approximations have a representation which allows us to "complexify" the parameter N.

In this paper, we obtain, as limits of these, analytic functions of two complex variables λ, z, which continue (in λ) and interpolate (in $z\sim\frac{1}{N}$) the given Schwinger functions without ultra-violet cut-off. (The removal of the volume cut-off using the Glimm-Jaffe-Spencer cluster expansion if $|\lambda|$ is sufficiently small does not seem to entail any essential difficulty.) We show that these analytic functions have an indefinitely derivable (in an angle) continuation to points of the form $(\lambda, z=0)$, if $|\lambda|$ is sufficiently small, and that their Taylor series at these points are Borel summable.

This property improves the relation between the "$\frac{1}{N}$ expansion" (known to be asymptotic [2]) and the function itself. It allows the construction of convergent approximations which depends only on the beginning of the series; these are "explicit" (as sums of Feynman graphs). Moreover it allows us to characterize the constructed interpolation among all analytic functions which coincide at $z=\frac{1}{N}$, ($N\in\mathbb{N}$) with the given Schwinger functions.

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