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General Quantum Measurements: Local Transition Maps

Richard Mercer
Wright State University, Dayton, OH 45435, USA

Abstract. On the basis of four physically motivated assumptions, it is shown that a general quantum measurement of commuting observables can be represented by a "local transition map," a special type of positive linear map on a von Neumann algebra. In the case that the algebra is the bounded operators on a Hilbert space, these local transition maps share two properties of von Neumann-type measurements: they decrease "matrix elements" of states and increase their entropy. It is also shown that local transition maps have all the properties of a conditional expectation of a von Neumann algebra onto a subalgebra except that their range is not restricted to the subalgebra. The notion of locality arises from requiring that a quantum measurement may be treated classically when restricted to the commutative algebra generated by the measured observables. The formalism established applies to observables with arbitrary spectrum. In the case that the spectrum is continuous we have only "incomplete" measurements.

1. Introduction

A long-standing problem in mathematical physics is the description of the quantum mechanical measurement of an observable with continuous spectrum. The lack of such a description is an obstacle to the development of an adequate theory of quantum stochastic processes, and to a complete understanding of non-relativistic quantum mechanics. This paper attempts to clear up one aspect of the problem, the effect of a measurement on the state of the system.

The following conventions and notation will be used in this paper. Hilbert spaces will be complex and separable with inner products conjugate linear in the first entry. $\mathscr{B}(\mathscr{H})$ will denote the bounded linear operators on a Hilbert space $\mathscr{H}$. $\mathscr{T}(\mathscr{H})$, $\mathscr{T}^2(\mathscr{H})$, and $\Sigma(\mathscr{H})$ will denote respectively the trace-class operators, Hilbert–Schmidt operators, and normal states on $\mathscr{B}(\mathscr{H})$. A state $\rho\in\Sigma(\mathscr{H})$ will be thought of interchangeably as a linear functional on $\mathscr{B}(\mathscr{H})$ or a trace-class operator in $\mathscr{B}(\mathscr{H})$. A subset of a topological vector space is "total" if its linear span is dense.

The following description, introduced by von Neumann [1], of the complete measurement of an observable with simple discrete spectrum is widely accepted.

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