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A Spectral Characterization of KMS States

J. De Cannière*

Department of Mathematics, University of California, Berkeley, CA 94720, USA

Abstract. Let ω be a state on a C^*-dynamical system $(\mathfrak{A}, \mathbb{R}, \alpha)$. For each of the following properties of ω: (1) ω is β-KMS with respect to α for some given β, $0 \leqq \beta < +\infty$, (2) ω is either a KMS state or a ground state, necessary and sufficient conditions are given involving only the spectral subspaces of $\mathfrak{A}$ associated with α. The results provide a new insight in the concept of passivity, introduced by W. Pusz and S. L. Woronowicz.

1. Introduction, Main Results, Preliminaries

Let $(\mathfrak{A}, \mathbb{R}, \alpha)$ be a C^*-dynamical system [8, Chapter 7], β a nonzero positive real number. A state ω on $\mathfrak{A}$ is said to be β-KMS *with respect to* α if for every pair $x, y \in \mathfrak{A}$ there exists a bounded continuous complex function F on the closure of the strip $D = \{z | 0 < \operatorname{Im} z < \beta\}$ that is holomorphic on D and has boundary values

$$\begin{aligned} F(t) &= \omega(y\alpha_t(x)) \\ F(t+i\beta) &= \omega(\alpha_t(x)y) \quad (t \in \mathbb{R}). \end{aligned} \tag{1.1}$$

This condition was introduced in the algebraic statistical theory of infinite quantum systems by R. Haag, N. M. Hugenholtz and M. Winnink [4] to provide a substitute for Gibbs states. The point of view that the KMS condition is to be satisfied by equilibrium states at inverse temperature β is supported by a fair amount of physically relevant mathematical evidence, a lot of which can be found in [2; 6]. Moreover the KMS condition plays a central role in the Tomita–Takesaki theory [10] and in non-commutative integration.

On the other hand, one of the main tools to study C^*-dynamical systems in general is the notion of spectral subspaces, the introduction of which in the theory of operator algebras is due to W. B. Arveson [1; 8, Chapter 8]. It is known, for instance, that α is completely determined if the spectral subspaces $R(-\infty, \lambda) \subset \mathfrak{A}$

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