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Star-Triangle Relations in the Exactly Solvable Statistical Models

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Abstract. A regular method for analysis of lattice spin models with a nearest neighbour interaction is proposed. Star-triangle relations in the form of functional equations are used. Parametric families of transfer matrices commuting due to star-triangle relations are constructed. The eigenvalues of transfer matrices as functions of the spectral parameter are shown to obey two functional equations. The solution of these equations for the maximal eigenvalue yields the partition function of the model. The method is applied for evaluation of the partition function of the critical Potts models, the Ising model, the Ashkin-Teller model equivalent to the eight-vertex model.

1. Introduction

The star-triangle transform has served for the analysis of the exactly solvable lattice statistical spin systems for a long time. It was briefly mentioned in the famous work by Onsager [1] and used in his article [2]. Star-triangle relations (STR) combined with dual transformation (DT) made it possible to determine the critical point of the Ising models on the triangular and honeycomb lattices [3–6]. Baxter and Enting evaluated the partition function of the Ising model using only STR [7]. STR were also utilized to construct the exact renormalization group equations for the Ising model on the triangular lattice [8, 9]. Mittag and Stephen [10] applied STR to the critical Potts models [11].

In the present paper we consider STR as functional equations. Their derivation is based on the Yang-Baxter equations of triangles (ET) [12–15]. The solutions of STR are used to construct parametric families of commuting transfer matrices (TM). These TM are ascertained to be normal operators. This allows us in principle to diagonalize the whole family by a single unitary transformation.

To find the eigenvalues of TM concerned one might use them to construct a Baxter-type TM [13] and then employ the generalized Bethe-ansatz technique. The latter was proposed by Baxter and elaborated to a standard procedure in [16, 17]. Unfortunately, the search of pseudo-vacuum that is the first step of this procedure

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