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The Global Existence of Yang–Mills–Higgs Fields in 4-Dimensional Minkowski Space

II. Completion of Proof*

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Abstract. In this paper we complete the proof of global existence of Yang–Mills–Higgs fields in 4-dimensional Minkowski space by showing that an appropriate norm of the solutions cannot blow up in a finite time. A key step in the proof is the demonstration that the L^∞ norm of the curvature is bounded *a priori*. Our results apply to any compact guage group and to any invariant Higgs self-coupling which is positive and of no higher than quartic degree.

I. Introduction

In this paper we shall complete the proof of global existence of Yang–Mills–Higgs (YMH) fields which we began in Ref. 1 (referred to hereinafter as paper 1). In paper 1 we established local existence, uniqueness and smoothness properties of YMH fields in the temporal gauge, improving earlier results [2, 3] for this system by essentially one order of differentiability. To extend the argument to a global existence proof we must show that the $(H_2 \times H_1 \times H_2 \times H_1)$ norm of $(A_i, \dot{A}_i, \phi, \dot{\phi})$ does not blow up in a finite time. To accomplish this we shall first derive an *a priori* bound on the norms $\|{}^{(4)}F(t)\|_{L^\infty}$ and $\|D\phi(t)\|_{L^\infty}$ where ${}^{(4)}F$ is the curvature of the Yang–Mills potential ${}^{(4)}A$ and $D\phi$ is the covariant gradient of the Higgs field ϕ. Given this estimate we can easily complete the proof by showing that a suitably defined higher order "energy" does not blow up.

To derive an estimate on the "curvatures" $({}^{(4)}F, D\phi)$ we adopt a method inspired by Jörgens' treatment of the non-linear wave equation [4]. We write an integral

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