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Proof of Confinement of Static Quarks in 3-Dimensional $U(1)$ Lattice Gauge Theory for all Values of the Coupling Constant*

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Abstract. We study the 3-dimensional pure $U(1)$ lattice gauge theory with Villain action which is related to the 3-dimensional $\mathbb{Z}$-ferromagnet by an exact duality transformation (and also to a Coulomb system). We show that its string tension α is nonzero for all values of the coupling constant g^2, and obeys a bound $\alpha \geqq \text{const} \cdot m_D \beta^{-1}$ for small ag^2, with $\beta = 4\pi^2/g^2$ and $m_D^2 = (2\beta/a^3)e^{-\beta v c_b(0)/2}$ (a = lattice spacing). A continuum limit $a \to 0$, m_D fixed, exists and represents a scalar free field theory of mass m_D. The string tension αm_D^{-2} in physical units tends to ∞ in this limit. Characteristic differences in the behaviour of the model for large and small coupling constant ag^2 are found. Renormalization group aspects are discussed.

1. Introduction and Discussion of Results

In this paper we will study the $\mathbb{Z}$-ferromagnet on a 3-dimensional cubic lattice $\Lambda \subsetneqq (a\mathbb{Z})^3$ of lattice spacing a. The spin variables $n(x)$ of the model are attached to the sites x of the lattice. They take values which are integer multiples of 2π. The partition function is

$$Z_\Lambda = \sum_{n \in (2\pi\mathbb{Z})^\Lambda} \exp L(n), \quad \text{with} \quad L(n) = -\frac{1}{2\beta} \int_x [\nabla_\mu n(x)]^2 . \tag{1.1}$$

We use the notations (e_μ = lattice vector of length a in μ-direction)

$$\int_x = a^3 \sum_{x \in \Lambda} ; \quad \nabla_{\pm\mu} n(x) = a^{-1}[n(x \pm e_\mu) - n(x)] . \tag{1.2}$$

β has dimension of a length, whereas n is dimensionless. Formula (1.1) must be supplemented by boundary conditions. We choose to immerse the system into an infinitely extended heat bath which is described by a massless free field theory, see Eqs. (2.3) of Sect. 2. [Formally, the partition function for the combined system is also given by Eq. (1.1), but the variables $n(x)$ are integrated over the reals outside

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