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Absence of Discrete Spectrum in Highly Negative Ions*

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Abstract. Let H_N be the Hamiltonian for the Coulomb system consisting of N particles of like charge in the field of a fixed point charge Z. We show that if the particles are bosons, then H_N has no discrete spectrum when $N \geqq N_0 = cZ^2$ for some constant c. If the particles are fermions, then H_N is bounded below uniformly in N. These results can be extended to molecules and to other power law potentials.

I. Introduction

Let H_N be the Hamiltonian

$$H_N(W, Z) = -\sum_{j=1}^{N} \Delta_j - \sum_{j=1}^{N} Zr_j^{-1} + \sum_{j<k} Wr_{jk}^{-1}. \tag{1a}$$

When $W=1, H_N$ is the Hamiltonian of N charged particles in the field of an infinitely heavy nucleus of charge Z. If these particles are fermions and $Z \geqq N+1$, so that $H_N(1,Z)$ is the Hamiltonian for a negative ion, it is known [1–3, 18] that H_N has only finitely many bound states. However, very little is known about the precise number of bound states. When $N=2$, Hill [4, 5] has shown that $H_2(1,1)$ which is the Hamiltonian for H^-, has precisely one bound state in the sector of natural parity; Grosse and Pittner [6] have shown that H^- has precisely three degenerate bound states in the sector of unnatural parity. Hill's results can be extended to show that H^{--} has no bound states [7], but Hill's techniques are unlikely to be suitable for N much larger than 3 or 4. All other methods known for estimating the number of bound states of multi-particle systems are either very specialized or very weak [8–10].

In this paper we show that for a system of N charged *bosons*, $H_N(W, Z)$ has no discrete spectrum when N is sufficiently large. Then the only possible bound states are eigenvalues imbedded in the continuum. Because our method of proof uses smoothing functions which need not leave a given symmetry subspace invariant,

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