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Time-Delay in Potential Scattering Theory

Some "Geometric" Results

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Abstract. Results on time-delay in potential scattering theory are given using properties of the generator of dilations ("geometric" method).

1. Introduction

The present paper is concerned with time-delay in potential scattering theory. Let $H_0 = -\Delta$ and $H = H_0 + V$ be the free and full Hamiltonian, respectively, in $\mathscr{H} = L^2(\mathbb{R}^n)$, with $V(x) = O(|x|^{-\beta})$, $\beta > 1$, as $|x| \to \infty$. Existence and completeness of the wave operators $W_\pm$ is well known. To define the time-delay, consider first an orthogonal projection P in $\mathscr{H}$. The probability of finding the state $e^{-itH}f$ in $P\mathscr{H}$ at time t is given by $\|Pe^{-itH}f\|^2$.

The total time spent in $P\mathscr{H}$ is given by

$$\int_{-\infty}^{\infty} \|Pe^{-itH}f\|^2 dt. \tag{1.1}$$

It is not obvious that this integral is finite. Finiteness is in many cases obtained for some f by proving local H-smoothness of P.

Let us briefly state the main problems in time-delay. Let P_r denote multiplication by the characteristic function for the ball $\{|x| < r\}$. Let $f \in \mathscr{H}$. $e^{-itH_0}f$ and $e^{-itH}W_- f$ are asymptotically equal as $t \to -\infty$. The difference of the times spent in $P_r\mathscr{H}$ by these two states is the time-delay for the ball $\{|x| < r\}$:

$$\Delta T_r(f) = \int_{-\infty}^{\infty} (\|P_r e^{-itH} W_- f\|^2 - \|P_r e^{-itH_0} f\|^2) dt. \tag{1.2}$$

As r tends to infinity, one expects a finite limit, at least for a dense set of $f \in \mathscr{H}$. The limit is the time-delay for f

$$\Delta T(f) = \lim_{r \to \infty} \Delta T_r(f). \tag{1.3}$$

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