Commun. Math. Phys. 82, 211–226 (1981)

Communications in Mathematical Physics

Invariant Subspaces of Clustering Operators. II

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Abstract. Clustering operators, when restricted to k-particle invariant subspaces, are shown still to cluster.

1. Introduction. The Formulation of the Main Theorem and the Plan of its Proof

This work is the continuation of [1], where the theorem was announced that for the clustering operator with sufficiently small values of the clustering parameter (see below), a) there exist invariant "k-particle" subspaces, and b) the restrictions of the clustering operator upon these subspaces are unitarily equivalent to some clustering operators. In [1] part a) of this theorem was proved. Here we prove part b) constructing this unitary equivalence explicitly. For the reader's convenience this work is almost self contained.

We consider the Hilbert space $l_2(\mathbf{C}_{\mathbb{Z}^\nu})$ of functions $f(T)$, $T\in\mathbf{C}_{\mathbb{Z}^\nu}$ where $\mathbf{C}_{\mathbb{Z}^\nu}$ is the family of all finite subsets (including the empty set) of $\mathbb{Z}^\nu$, $\nu \geqq 1$. The operator A in $l_2(\mathbf{C}_{\mathbb{Z}^\nu})$, defined as

$$(Af)(T)=\sum_{T'\in\mathbf{C}_{\mathbb{Z}^\nu}} a_{T,T'}f(T') \quad T\in\mathbf{C}_{\mathbb{Z}^\nu} \tag{1.1}$$

and commuting with unitary group $\{U_t, t\in\mathbb{Z}^\nu\}$ of translations in $l_2(\mathbf{C}_{\mathbb{Z}^\nu})$:

$$(U_tf)(T)=f(T-t), \quad T\in\mathbf{C}_{\mathbb{Z}^\nu}, \quad t\in\mathbb{Z}^\nu, \tag{1.2}$$

$T-t=\{x_i-t, i=1,2\dots,|T|\}$, if $T=\{x_i, i=1,2,\dots|T|\}$ and $x_i\in\mathbb{Z}^\nu$, $i=1,2\dots,|T|$, is called *clustering* if its matrix elements $a_{T,T'}$ satisfy the *cluster expansion*

$$a_{T,T'}=\sum_{k\geqq 1}\sum_{\{\tau_1,\dots,\tau_k\}}\omega_k(\tau_1,\dots,\tau_k). \tag{1.3}$$

Let $Y_0=\{0\}\times\mathbb{Z}^\nu\subset\mathbb{Z}^{\nu+1}$ be a zero-time slice of $\mathbb{Z}^{\nu+1}$ and $Y_1=\{1\}\times\mathbb{Z}^\nu\subset\mathbb{Z}^{\nu+1}$. Let π_0: $\mathbb{Z}^\nu\to Y_0$ and π_1: $\mathbb{Z}^\nu\to Y_1$ be identity maps. We define τ_i to be a pair (T_i,T_i') of subsets of $\mathbb{Z}^\nu$. We shall often identify τ_i with the subset $\pi_0(T_i)\cup\pi_1(T_i')$ of $Y_0\cup Y_1$. The summation in (1.3) is over all partitions $(\tau_1,\dots,\tau_k)$ of $\pi_0(T)\cup\pi_1(T')\equiv(T,T')$.

0010-3616/81/0082/0211/$03.20