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On the Free Boson Gas in a Weak External Potential

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Abstract. We calculate the equation of state and the barometric formula for a d-dimensional free boson gas in a weak external field of power form. We find that the condensate has a complicated structure in two dimensions.

1. Introduction

In this paper we investigate the phenomenon of condensation in a non-interacting boson gas in the presence of an external field, confined in a region of d-dimensional Euclidean space by a container with hard walls. It is well known that, in the absence of an external field, condensation cannot occur unless the dimension d is greater than two. The result that an external field can induce condensation in one dimension was announced in [1]. Here we prove that if the external field comes from a scaled potential which is a positive power α of one coordinate, then the critical properties are determined by an effective dimension $d + 2/\alpha$; using this rule, the critical properties can be read-off from the list given for an ideal boson gas in arbitrary dimension by Ziff, Uhlenbeck and Kac [2].

The use of a scaled potential requires some explanation. It has been known for about fifty years that, in order to display a phase transition in a sharp way mathematically, it is necessary to pass to the thermodynamic limit. In the case of a free boson gas in a cubical container C_a with side of length a, this is the limit in which a increases without bound in comparison with the thermal wave-length λ while the density remains fixed. On the other hand, since we are interested in the way in which the condensation phenomenon is modified by the presence of an external field, we must ensure that the effect of the potential $V(y)$ is not so great as to destroy the thermodynamic behaviour. This suggests that we should investigate the case in which $V(y)$ varies slowly as y ranges over the container C_a; to express this mathematically, we write $V(y)$ as a function ϕ of y/a and require that ϕ satisfies some local condition of slow variation. But the thermodynamic limit has to be taken, so the local condition on ϕ must be replaced by a global condition of slow variation. We expect results in the thermodynamic limit (under these

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