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Convergence to Equilibrium of the Stochastic Heisenberg Model

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Abstract. The stochastic Heisenberg model is a probabilistic model of the time evolution of a classical Heisenberg ferromagnet. It is proved that the stochastic process converges to equilibrium at sufficiently high temperatures, and that the equilibrium state is a Gibbs state of the Hamiltonian possessing the global Markov property. The principal technique employed is the expansion of observables on the state space into Laplace series.

I. Introduction

The stochastic Heisenberg model was introduced by William Faris in [1] as a probabilistic model of the time evolution of a classical Heisenberg ferromagnet. He constructed an infinite-dimensional Markov diffusion process whose state space is a countable product of spheres, and which has two components. The first is a random motion, the product of Brownian motions on the individual spheres. The second is a deterministic drift generated by a vector field containing the interaction terms. The latter is formally the negative of the gradient of the Hamiltonian, multiplied by the inverse temperature.

In a second paper [2] the same author studied the equilibrium state of the model and proved that it is unique and analytic in the inverse temperature for sufficiently high temperatures. This was obtained by expanding the equilibrium state into a series of Rayleigh–Schrödinger type around a product measure, and proving convergence of the series in an appropriate norm.

The main theorem of the present paper (Theorem 1 of Sect. III) establishes convergence of this process with arbitrary initial state to the equilibrium state at high temperature, for a large class of models of the type studied by Faris. The principal technique we employ is the expansion of functions on the state space into series of eigenfunctions of the infinite-dimensional Laplacian (Laplace series.) We include an existence theorem for the process of a different sort than Faris's. We also give a discussion of the relationship between the equilibrium state of the stochastic model and the Gibbs state of the Hamiltonian and show how the global Markov property of the equilibrium state may be deduced from Faris's uniqueness theorem.

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