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A Mass Zero Cluster Expansion

Part 2. Convergence*

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Abstract. Convergence is proven for the mass zero cluster expansion presented in Part 1 of this paper. An indication is given of changes necessary to treat the more difficult $\lambda(\vec{\nabla}\phi)^4$ model and the lattice dipole gas.

9. Counting I

In these sections we will be concerned with enumerating the terms in the cluster expansion in a way suitable for estimation. The complexities are largely notational, and due to the need to consider a number of different cases, there are no real difficulties. Then too, this is a new type of cluster expansion, and its "standard tricks" have to be invented. We will try to give motivation for a number of the avenues taken.

9.A. Representation 1

We here present the representation (labelling) of a single term in the cluster expansion, basically as developed in Sect. 8. In later subsections we will find alternate representations more useful for computation.

A term in the cluster expansion is determined by giving

1) A finite sequence

$$(i_1, x_1), (i_2, x_2), \ldots$$

with

$$(i_s, x_s) < (i_{s+1}, x_{s+1}) \tag{9.A.1}$$

in the order (8.A.1) of [2].

2) A mapping

$$(i_s, x_s) \to (\alpha_1(s), \alpha_2(s), \alpha_3(s), \alpha_4(s)). \tag{9.A.2}$$

Clearly the elements in the sequence are just the (i, x) not mapped into N by T, and (9.A.2) is just the mapping T. There are compatibility conditions we are omitting so that not all terms we have specified are nonzero. The order of the four

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