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Jump Discontinuities of Semilinear, Strictly Hyperbolic Systems in Two Variables: Creation and Propagation

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Abstract. The creation and propagation of jump discontinuities in the solutions of semilinear strictly hyperbolic systems is studied in the case where the initial data has a discrete set, $\{x_i\}_{i=1}^n$, of jump discontinuities. Let S be the smallest closed set which satisfies:
(i) S is a union of forward characteristics.
(ii) S contains all the forward characteristics from the points $\{x_i\}_{i=1}^n$.
(iii) if two forward characteristics in S intersect, then all forward characteristics from the point of intersection lie in S.
We prove that the singular support of the solution lies in S. We derive a sum law which gives a lower bound on the smoothness of the solution across forward characteristics from an intersection point. We prove a sufficient condition which guarantees that in many cases the lower bound is also an upper bound.

1. Introduction

This paper is devoted to the study of the regularity of locally bounded solutions to strictly hyperbolic semilinear first order systems in one space variable. That is, we study $u \in L^\infty_{\mathrm{loc}}(\Omega)$ satisfying

$$A_0(x, t)\partial_t u - A_1(x, t)\partial_x u = G(x, t, u) \tag{1.1}$$

where the A_i are smooth $m \times m$ complex matrix-valued functions. We suppose that the system is strictly hyperbolic, that is, $\det A_0 \neq 0$ and the equation $\det(A_0 - \lambda A_1) = 0$ has m distinct real roots, $\{\lambda_i\}_{i=1}^m$, for all $\langle x, t \rangle$ under consideration. We study solutions on R_T, the open trapezodial region bounded above and below by the lines $t = T$, $t = 0$, on the left by a characteristic of maximal speed, and on the right by a characteristic of minimal speed. We let $I_{\bar{t}} = R_T \cap \{\langle x, t \rangle \mid t = \bar{t}\}$.

If $u \in L^\infty(R_T)$ satisfies (1.1) in the sense of distributions, then u is weakly continuous on $[0, T]$ with values in L^∞ in the sense that for any fixed $\tilde{t} \in (0, T)$ and

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