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Some Global Solutions of the Yang–Mills Equations in Minkowski Space†

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Abstract. A class of global solutions of the Yang–Mills equations whose Cauchy data depend on a pair of arbitrary functions is constructed. The asymptotic propagation of the energy in space-time is studied. The same results are valid if the Yang–Mills field is coupled to a scalar field.

1. Introduction

This paper is concerned with solutions of the classical Yang–Mills equations in the whole of Minkowski space-time. Although one would like to exhibit such global solutions by explicit formulas, the best representation one could envisage for a general solution would be its exhibition as an infinite series or as the result of a limiting procedure. Since the equations essentially comprise a hyperbolic system of partial differential equations, it is natural to consider the Cauchy problem: do arbitrary data at a finite time (say $t = 0$) determine a unique solution at all later and earlier times? In this paper we restrict our considerations to data which have finite energy and whose potentials vanish as $|x| \to \infty$.

It should be noted that the Yang–Mills equations are entirely different in Euclidean space. They form an elliptic, rather than a hyperbolic, system. In particular, the condition that a solution be square-integrable in space *and* imaginary time is a severe one. It makes the instantons form a class depending only on a few parameters.

In Sect. 2 we use a limiting procedure to construct a family of global solutions to the Yang–Mills equations (YM) if the gauge group is $SU(2)$. The solutions depend on two arbitrary functions of one variable. A substitution of the Polyakov–t'Hooft type specifies the direction of the gauge potentials A^μ and expresses their amplitudes in terms of a single scalar function $\alpha(|x|, t)$. The equations are thereby reduced to a single scalar nonlinear wave equation with singular coefficients. Except for the condition of finite energy, the Cauchy data for this

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