Commun. Math. Phys. 80, 485–516 (1981)

Communications in Mathematical Physics

Finite Time Analyticity for the Two and Three Dimensional Kelvin-Helmholtz Instability

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Abstract. The well-posed property for the finite time vortex sheet problem with analytic initial data was first conjectured by Birkhoff in two dimensions and is shown here to hold both in two and three dimensions. Incompressible, inviscid and irrotational flow with a velocity jump across an interface is assumed. In two dimensions, global existence of a weak solution to the Euler equation with such initial conditions is established. In three dimensions, a Lagrangian representation of the vortex sheet analogous to the Birkhoff equation in two dimensions is presented.

1. Introduction

A velocity discontinuity (vortex sheet) in an ideal incompressible fluid is subject to the Kelvin-Helmholtz instability [see Birkhoff (1962) and Saffman and Baker (1979) for a general introduction]. A simple illustration is provided by a flow with uniform velocity U in the x-direction above the (x, y) plane and with the same velocity in the opposite direction below this plane. Such a motion constitutes a stationary but unstable solution to the equations of fluid dynamics (see e.g. Chandrashekar, 1961). When a slight disturbance preserving the irrotationality of the flow outside the interface is considered, a linear analysis indicates that the amplitude of the k-Fourier mode of the interface corrugation increases exponentially in time at the rate $|k \cdot U|$. The linear problem therefore requires analytic initial data to be well posed and will generally be so only for a finite time. Birkhoff (1962) conjectures that the nonlinear problem with analytic initial data is well posed at least for a finite time. Richtmyer and Morton (1967) make a similar conjecture for piecewise analytic data.

The present paper is devoted to the nonlinear problem with analytic initial data. We shall not, as is mostly the case in studies of Kelvin-Helmholtz instability,

* This work was performed while C.B. was visiting the Dept. de Mathématiques, Nice

0010-3616/81/0080/0485/$06.40