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Interacting Quantum Fields Around a Black Hole

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Abstract. If one studies interacting fields on a black hole background using ordinary Feynman diagrams, one is faced with a problem of what to do with lines that cross the horizon. To avoid this difficulty a formulation is developed which can be expressed graphically in terms of a new class of diagram with external lines only at infinity. This formalism is applied to study the question of whether spontaneously broken symmetry would be restored near the black hole. It is also used to show that a black hole can emit more particles than antiparticles even in theories where the particle number is locally conserved by a global $U(1)$ symmetry.

1. Introduction

The theory of non-interacting quantum fields in the vicinity of a black hole is now well understood in the semi-classical approximation in which the gravitional field of the black hole can be regarded as a fixed stationary background. One finds that the black hole creates and emits particles as if it were a hot body [1] with a temperature proportional to the surface gravity of the black hole [2]. (In the case of an uncharged non-rotating black hole,

$$T=(8\pi M)^{-1}$$

in units in which $G=c=\hbar=k=1$, where M is the mass of the black hole.) One can understand this emission heuristically as follows: The Uncertainty Principle implies that "empty" spacetime is filled with closed loops of particles. One can visualise one of these as corresponding to a particle-antiparticle pair which appear together, move apart and them come together again and annihilate each other. They cannot continue to exist as "real" particles because in that case they would both have positive energy which would violate the conservation of energy. However when a black hole is present, one of the pair can tunnel throught the horizon into the interior where there are particle states which have negative energy with respect to infinity. This allows the other particle to escape to infinity as a real particle

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