Commun. Math. Phys. 80, 153–179 (1981)

Communications in
Mathematical
Physics

A General Lee–Yang Theorem for One-Component and Multicomponent Ferromagnets

Elliott H. Lieb[1★] and Alan D. Sokal[2★★]

1 Departments of Mathematics and Physics, Princeton University, Princeton, NJ 08544, USA
2 Department of Physics, Princeton University, Princeton, NJ 08544, USA

Abstract. We show that any measure on $\mathbb{R}^n$ possessing the Lee–Yang property retains that property when multiplied by a ferromagnetic pair interaction. Newman's Lee–Yang theorem for one-component ferromagnets with general single-spin measure is an immediate consequence. We also prove an analogous result for two-component ferromagnets. For N-component ferromagnets ($N \geqq 3$), we prove a Lee–Yang theorem when the interaction is sufficiently anisotropic.

1. Introduction

The Lee–Yang theorem on the zeros of the partition function is an important tool in the rigorous study of phase transitions in lattice spin systems [1]. In addition, it has applications to the proof of existence of the infinite-volume limit [2] and of a mass gap [3, 4], and to the proof of correlation inequalities [5, 6] and inequalities for critical exponents [4, 7, 8].

In this paper we shall give a new proof of a generalized Lee–Yang Theorem. Our methods lead to an essentially complete result for one-component and two-component (classical) ferromagnets with quite general single-spin measures. We have also some promising partial results for N-component ferromagnets ($N \geqq 3$). We end the paper with some conjectures.

Consider, for purposes of orientation, the model of one-component "spins" ϕ_i defined by the partition function

$$Z = \int \exp\left[\sum_{i,j=1}^{n} J_{ij}\varphi_i\varphi_j + \sum_{i=1}^{n} h_i\varphi_i\right]\prod_{i=1}^{n} dv_i(\varphi_i). \tag{1.1}$$

Here the dv_i are suitable probability measures on the real line; the pair interaction coefficients J_{ij} are nonnegative ("ferromagnetic"); and the magnetic fields h_i are allowed to take arbitrary complex values. The Lee–Yang theorem then states

★ Research supported in part by NSF grant PHY 78-25390 A01
★★ Research supported in part by NSF grant PHY 78-23952

0010-3616/81/0080/0153/$05.40