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Schrödinger Operators With Magnetic Fields

III. Atoms in Homogeneous Magnetic Field

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Abstract. We prove a large number of results about atoms in constant magnetic field including (i) Asymptotic formula for the ground state energy of Hydrogen in large field, (ii) Proof that the ground state of Hydrogen in an arbitrary constant field has $L_z = 0$ and of the monotonicity of the binding energy as a function of B, (iii) Borel summability of Zeeman series in arbitrary atoms, (iv) Dilation analyticity for arbitrary atoms with infinite nuclear mass, and (v) Proof that every once negatively charged ion has infinitely many bound states in non-zero magnetic field with estimates of the binding energy for small B and large L_z.

1. Introduction

This is the third paper in our series on Schrödinger operators with magnetic field concentrating especially on the case of constant magnetic field where we normally use the gauge

$$\mathbf{a} = \tfrac{1}{2}(\mathbf{B}_0 \times \mathbf{r}) \,. \tag{1.1}$$

In this paper we consider primarily the physically important case of Coulomb forces and constant B. There turn out to be a number of previously undiscovered phenomena of mathematical and/or physical interest. This is, in part, because of the dearth of previous mathematical literature on the subject and, in part, because the natural units of B are so large (about 10^9 Gauss) that theoretically interesting efforts at large field cannot be seen in the laboratory with present techniques. The natural unit of B is $B^* = \frac{1}{2}mc^2(\alpha^2/\mu_B) = 2.35 \times 10^9$ Gauss; μ_B the Bohr magneton and $\frac{1}{2}mc^2\alpha^2$ the binding energy of Hydrogen. In a field of size B^*, the Landau energy exactly equals the ground state energy.

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