

On the Spherically Symmetric Gauge Fields

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Abstract. The spherically symmetric gauge fields with a compact gauge group over 4-dimensional Minkowski space are determined completely. Expressions for the gauge potentials of these fields are obtained.

Part I. Construction of Spherically Symmetric Gauge Fields

1. Introduction

Spherical symmetry is one of the most important symmetry in nature. In a field theory the spherically symmetric fields are very useful for understanding the theory itself. For example, the important role of Schwarzschild solution in Einstein's gravitation theory is well-known. For the gauge theories, the fields with spherical symmetry are also very interesting. It is known that two gauge fields are equivalent if they are related by a gauge transformation. Consequently, in the construction of spherically symmetric gauge fields we have to consider the effect of gauge transformations. Spherically symmetric potentials of SU_2 gauge fields were firstly considered by Wu and Yang [1]. Now the complete classification of spherically symmetric SU_2 gauge fields is known [2]. Some spherically symmetric SU_3 gauge potentials were studied by several authors [3–5]. For the general gauge groups the problem of determining spherically symmetric field may be reduced to solve some system of partial differential equations by using Lie derivatives [6]. But, the results in [6] are local in character and no formulas for the gauge potential were obtained. The same problem was treated in a different way in a previous paper [7], an algebraic method for determining all spherically symmetric gauge fields was proposed. However, in this paper the expressions of the gauge potentials were given only for some special cases and the proof of the general theorem was not complete yet. It should be noted that some special form of spherically symmetric gauge fields were pointed in [8].

In the present paper we develop this method and give a complete determination of general spherically symmetric gauge fields for any compact gauge