Commun. Math. Phys. 79, 47–51 (1981)

Communications in Mathematical Physics

The Energy and the Linear Momentum of Space-Times in General Relativity

Richard Schoen and Shing-Tung Yau*
The Institute for Advanced Study, Princeton, NJ 08540, USA

Abstract. We extend our previous proof of the positive mass conjecture to allow a more general asymptotic condition proposed by York. Hence we are able to prove that for an isolated physical system, the energy momentum four vector is a future timelike vector unless the system is trivial. Furthermore, we allow singularities of the type of black holes.

In our recent solution of the positive mass conjecture, we assumed that the initial data on the three-dimensional manifold is asymptotically spherical up to terms of quadratic decay. This was the classical formulation of the conjecture. However, in a recent paper, York [5] pointed out that physically it would be very desirable to push this classical formulation to a more general setting. York's condition seems to be the most general condition that may arise from an isolated physical system. In this paper, we prove the positive mass conjecture assuming merely York's condition. Our method depends on the construction of a new initial data set which verifies our previous asymptotic condition and which has approximately the same energy. Thus the positivity of the energy in the most general setting follows from the result of our previous paper.

Let us now recall York's formulation of the positive mass conjecture. An initial data set for a space-time consists of a three-dimensional manifold N, a positive definite metric g_{ij}, a symmetric tensor h_{ij}, a local mass density μ, and a local current density J^i. The constraint equations which determine N as a space-like hypersurface in a space-time with second fundamental form h_{ij} are given by

$$\mu = \frac{1}{2}\left[R - \sum_{i,j} h^{ij}h_{ij} + \left(\sum_i h_i^i\right)^2\right]$$

$$J^i = \sum_j \nabla_j\left[h^{ij} - \left(\sum_k h_k^k\right)g^{ij}\right]$$

* Supported in part by an NSF grant

0010-3616/81/0079/0047/$01.00