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Absence of Singular Continuous Spectrum for Certain Self-Adjoint Operators

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Abstract. We give a sufficient condition for a self-adjoint operator to have the following properties in a neighborhood of a point E of its spectrum:
a) its point spectrum is finite;
b) its singular continuous spectrum is empty;
c) its resolvent satisfies a class of a priori estimates.

Notations, Definitions, and Main Theorem

Let H be a self-adjoint operator on a Hilbert space $\mathscr{H}$. We will denote by $\mathscr{H}_n(n\in\mathbf{Z})$ the Hilbert space constructed from the spectral representation for H with the scalar product:

$$(\Phi|\Psi)_n=\int(\lambda^2+1)^{n/2}(\Phi|P_H(d\lambda)\Psi).$$

For functions $P\in L^\infty(\mathbf{R})$, P_H will denote the associated operator given by the usual functional calculus.

$P_H(E,\delta)$ will denote the spectral projection for H onto the interval $(E-\delta,E+\delta)$. P_H^p and P_H^c will denote the spectral projectors respectively onto the point spectrum and the continuous spectrum of H; $\sigma_c(H)=\mathbf{R}/\{E\in\mathbf{R}|E$ is an eigenvalue of $H\}$.

If A is a self-adjoint operator and $D(A)\cap D(H)$ is dense in $\mathscr{H}$, $i[H,A]$ will denote the symmetric form on $D(A)\cap D(H)$ given by

$$(\Phi|i[H,A]\Psi)=i\{(H\Phi|A\Psi)-(A\Phi|H\Psi)\}$$

for $\Psi,\Phi\in D(A)\cap D(H)$. If this form is bounded below and closeable, $i[H,A]^0$ will denote the self-adjoint operator associated to the closure [1].

1. Definition. Let H be a self-adjoint operator on a Hilbert space with domain $D(H)$; a self-adjoint operator A is a conjugate operator for H at a point $E\in\mathbf{R}$ if and only if the following conditions hold:

(a) $D(A)\cap D(H)$ is a core for H.

(b) $e^{+iA\alpha}$ leaves the domain of H invariant and for each $\Psi\in D(H)$

$$\sup_{|\alpha|<1}\|He^{+iA\alpha}\Psi\|<\infty.$$

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