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Superspaces and Supersymmetries*

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Abstract. A theory of graded Banach modules over a Banach–Grassmann algebra is developed and applied to differential geometry of super-manifolds. The explicit structure of superspaces carrying Poincaré supersymmetry and extended supersymmetry, including central charges, is described.

1. Introduction

Recently Alice Rogers [1] introduced a concept of supermanifold which seems to have some advantages over previous approaches. The idea is to fix a Grassmann algebra B_L (the number of odd generators L being possibly infinite), equip it with a Banach norm, and then work with Banach manifolds exploiting at the same time the Grassmann algebra structure. The present paper is inspired by this idea with the aim of improving a few unsatisfactory aspects of [1] and showing on explicit examples of physical interest how this theory works in practice.

After analysing the mathematical structure involved in [1] we have found that there are only few properties of B_L which really matter. Therefore we have introduced a concept of a Banach–Grassmann algebra Q (Sect. 3) which, in general, need not have a denumerable set of odd generators. The most important property of Q (apart of the fact that Q is Z_2-graded: $Q = Q_0 \oplus Q_1$) is its self-duality (see Definition 3.1a). So, we work with the category of graded Banach modules over Q, the fundamental principle being that of Q_0-linearity of all linear maps. Once the category is fixed and fundamental principle taken into account, all the theory becomes simple and quite elegant. In particular the tangent bundle of a supermanifold has exactly the same meaning as in ordinary differential geometry. Tangent vectors are tangent to one-parameter curves, and vector fields generate flows along their integral curves. Appealing to derivations is possible but not necessary. "Odd" vectors are well-defined geometrical objects, and can be constructed at any given point from tangent vectors in a canonical way, their only

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