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Lattice Systems with a Continuous Symmetry

II. Decay of Correlations

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Abstract. We consider perturbations of a massless Gaussian lattice field on $\mathbb{Z}^d, d \geqq 3$, which preserves the continuous symmetry of the Hamiltonian, e.g.,

$$-H = \sum_{\langle x,y \rangle} (\phi_x - \phi_y)^2 + T(\phi_x - \phi_y)^4, \ \phi_x \in \mathbb{R}.$$

It is known that for all $T > 0$ the correlation functions in this model do not decay exponentially. We derive a power law upper bound for all (truncated) correlation functions. Our method is based on a combination of the log concavity inequalities of Brascamp and Lieb, reflection positivity and the Fortuin, Kasteleyn and Ginibre (F.K.G.) inequalities.

I. Introduction

In this paper, we consider the same model of an anharmonic crystal as in [5] (part I of this series):

$$-\beta H = \sum_{\langle x,y \rangle} [(\phi_x - \phi_y)^2 + T(\phi_x - \phi_y)^4]$$

where $\langle x, y \rangle$ indicates that sum is over nearest neighbors in $\mathbb{Z}^d$. For $T = 0$, this is a massless Gaussian model and it is known that the correlation functions $\langle \phi_0, \phi_x \rangle$ and $|\langle \nabla_0^e \phi \nabla_x^e \phi \rangle|$ are not summable over the lattice (where $\nabla_x^e \phi = \phi_{x+e} - \phi_x$, e is a unit vector).

The question that we try to answer is: what is the decay of the correlations when $T > 0$?

Using the Brascamp and Lieb inequalities and some refinements of them,

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