Commun. Math. Phys. 76, 277–287 (1980)

Communications in
Mathematical
Physics


# On Bounded Solutions of a Classical Yang-Mills Equation

Michael Renardy

Institut für theoretische Physik, Universität Stuttgart, Pfaffenwaldring 57, D-7000 Stuttgart 80, Federal Republic of Germany

**Abstract.** We discuss bounded solutions of the equation

$$r^2\left(\frac{\partial^2 u}{\partial r^2}+\frac{\partial^2 u}{\partial t^2}\right)=u^3-u$$

in the halfspace $r>0$. All solutions depending only on $t/r$ are characterized topologically. Then we prove the existence of infinite dimensional manifolds of $t$-periodic as well as nonperiodic solutions which are small in a suitable norm.

## 0. Introduction

It was shown recently by Glimm and Jaffe [1] that multimeron solutions to the classical SU(2) Yang-Mills field equations in Euclidean space are characterized by the following singular elliptic boundary value problem:

$$\begin{gathered}r^2\left(\frac{\partial^2 u}{\partial r^2}+\frac{\partial^2 u}{\partial t^2}\right)=u^3-u \qquad t\in R, \qquad r>0, \\ \lim_{r,t\to\infty} u(r,t)=1, \qquad u(0,t)=(-1)^i \quad \text{for} \quad t_i<t<t_{i+1}(i=0,1,\ldots,2n),\end{gathered} \tag{0.1}$$

where $-\infty=t_0<t_1<\ldots<t_{2n-1}<t_{2n}<t_{2n+1}=\infty$.

Jonsson et al. proved in [2] that this boundary value problem has at least one solution for every choice of the $t_i$. In this paper we investigate some kinds of *bounded solutions to (0.1), which satisfy different boundary conditions.*

We first prove (Sect. 1) that a bounded solution of (0.1) which has a continuous extension to the $t$-axis except for a countable number of points must satisfy $|u|\leqq 1$ in the whole half-plane and cannot be positive everywhere, unless it is constant.

The special solutions which we discuss then are of two different types. In Sect. 2 we are concerned with solutions depending only the independent variable $\frac{t}{r}$, for which (0.1) is reduced to an ordinary differential equation; in Sects. 3 and 4 we discuss solutions which are "small" in a suitable norm.

0010-3616/80/0076/0277/$02.20