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"Time Inversion" and Mobility of Many Particle Systems

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Abstract. The unitary operations which can be generated on many particle states in non-relativistic quantum mechanics are discussed. These operations depend on an arbitrary external field which is in the experimenter's control, whereas the pairwise potential of interaction between the particles is fixed. The various kinds of systems of N identical particles interacting via the potentials $V_I = \sum_{k,j} r_{kj}^p w(r_{kj})$ are studied. For every system in question, the semigroup spanned by evolution transformations is proved to contain all the unitary operators in the Hilbert space of states. In particular, it is shown that the natural evolution operation can be reversed by a certain prescribed sequence of maneouvres involving only external fields.

1. Introduction

In quantum theories some non-obvious truths concerning the mathematical formalism are usually taken for granted. Thus, pure states are represented by vectors in a Hilbert space $\mathscr{H}$. Observables are self-adjoint operators. However, some questions concerning the "economy" of the mathematical language arise. Is every vector $\psi \in \mathscr{H}$ indeed necessary to describe some physical state which can be effectively created? Moreover, can one prove that to every self-adjoint operator there corresponds some effective measuring prescription? Is'nt it so, to the contrary, that physically essential observables are only the customarily considered quantities like energy, momentum, spin etc.? A similar question can be raised about the dynamical aspect of the theory. Evolution transformations of a quantum system are represented by unitary operators. However, can all the unitary operators be interpreted as the dynamical operations?

In spite of their apparent obviousness these questions are non trivial even in the one particle theory [1, 2]. They become more important in the theory of composite systems, due to interrelations with the Einstein-Podolsky-Rosen paradox and the corresponding distinction between the states of the first and the second kind [3]. The arguments concerning the particle production and the joint

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