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A Connection Between ν-Dimensional Yang–Mills Theory and $(\nu - 1)$-Dimensional, Non-Linear σ-Models

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Abstract. We study non-linear σ-models and Yang–Mills theory. Yang–Mills theory on the ν-dimensional lattice $\mathbb{Z}^\nu$ can be obtained as an integral of a product over all values of one coordinate of non-linear σ-models on $\mathbb{Z}^{\nu-1}$ in random external gauge fields. This exhibits two possible mechanisms for confinement of static quarks one of which is that clustering of certain two-point functions of those σ-models implies confinement of static quarks in the corresponding Yang–Mills theory. Clustering is proven for all one-dimensional σ-models, for the $U(n) \times U(n)$ σ-models, $n = 1, 2, 3, \ldots$, in two dimensions, and for the $\mathrm{SU}(2) \times \mathrm{SU}(2)$ σ-models for a large range of couplings $g^2 \gtrsim O(\nu)$. Arguments pertinent to the construction of the continuum limit are discussed. A representation of the expectation of Wilson loops in terms of expectations of random surfaces bounded by the loops is derived when the gauge group is $\mathrm{SU}(2)$, $U(n)$ or $O(n)$, $n = 1, 2, 3, \ldots$, and connections to the theory of dual strings are sketched.

1. Connections Between σ-Models and Yang–Mills Theory: Description of the Basic Ideas

In this paper we propose to study ν-dimensional (lattice) Yang–Mills theory, in terms of $(\nu - 1)$-dimensional (lattice) σ-models in random external gauge fields. Our main results are the ones described in the abstract. One can also apply our scheme to the study of $\mathbb{Z}(2)$ lattice gauge theories in three and four dimensions and relate them to a two-dimensional Ising model with random couplings in one direction, but this is not studied in this paper. Furthermore, we study a weak coupling limit of Yang–Mills theory relating this theory to *linear* σ-models in an external gauge field, in one dimension less. It appears to provide a *lower bound* on the confining potential—i.e. an *upper bound* on expectations of Wilson loop observables—with a convergent continuum limit. This bound is rigorous in the

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