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$1/n$ Expansion for a Quantum Field Model*

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Abstract. A nonperturbative study of the $1/n$ expansion in Euclidean Quantum Field Theory is started. The expansion is shown to be asymptotic to the vacuum energy of the $(\boldsymbol{\phi}^2)^2_2$ model, for arbitrary coupling constant.

1. Introduction

The $1/n$ expansion has recently aroused a great deal of interest among field theorists (for references see [1]). A rigorous, nonperturbative study of this expansion was started in [1], where the lattice nonlinear σ-model was considered. The techniques of [1] can also be applied to the $\lambda(\boldsymbol{\phi}^2)^2$ lattice field theory (in fact this theory is much easier to control; one can prove the estimates uniformly in λ as $\lambda\to\infty$). However, it is not possible to pass to the continuum limit using these methods. The reason for this is that the random walk expansions which played a central role in the estimates produce a divergence as ε, the lattice spacing, tends to zero. The source for the divergence lies in the fact that we perturbed in the off-diagonal part of Δ (the lattice Laplacean), which perturbation becomes singular as $\varepsilon\to 0$.

In the present paper we develop a different method to examine the $1/n$ expansion for the two dimensional (continuum) $(\boldsymbol{\phi}^2)^2$ quantum field theory. In particular we will show that the expansion is asymptotic to the pressure (vacuum energy) for arbitrary large coupling constant. The idea is to derive a closed form expression for the remainder in terms of Schwinger functions of suitable type. These can then be bounded using chessboard estimates and Hölder's inequality. We present two ways to generate the expansion with a remainder, one using a "dual" representation of the model in terms of a complex nonlocal measure, the other using the ϕ-representation.

The techniques of this paper can be used to derive the expansion for Schwinger functions with the remainder given again in terms of Schwinger functions. The existing cluster expansions can be applied to construct these for small coupling λ;

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