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A Spin-Statistics Theorem for Composites Containing both Electric and Magnetic Charges

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Abstract. The present paper states and proves an asymptotic spin-statistics theorem for composites consisting of electrically and magnetically charged particles. We work in the framework of a nonrelativistic theory, taking as the classical configuration space a $U(1)$ bundle over the space of physical configurations, and as the quantum hilbert space the homogeneous square integrable functions on that bundle. The theorems are proved using a formalism we develop here for treating "gauge spaces" – $U(1)$ bundles with connections; in particular, two products related to tensor products of vector bundles prove to be extremely useful in displaying the structure of the gauge spaces that naturally arise in this theory.

1. Introduction

This paper is a sequel to one in which we formulated a first-quantized theory of a system of electric and magnetic charges interacting (non-relativistically) through their instantaneous force-fields [1]. As the culmination of that paper, which we call here "*I*", we described – in special cases – the mechanisms by which both the *spin-type* (integer or half-integer) of a dyonic composite *and concomitantly* its asymptotic *statistics* (even- or oddness of the wave function under dyon interchange) can be in effect the reverse of those given by the usual combination rules (see also [2, 3, 9]). We then sketched, and promised to prove, a more general result which first would clarify in what sense dyons do behave in the asymptotic limit just like ordinary particles whose degrees of freedom split up into internal and external ones so that their spin and statistics can be reliably defined, and second would confirm from this point of view that the above mentioned reversals occur precisely when they "ought to". The present paper undertakes these tasks.

In order to express and prove our results most clearly, we have to redescribe the bundle of *I* in a coordinate free way, which unfortunately relies on a much larger body of mathematics than figures in I. Rather than obscure the simple

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