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The Ground State Energy per Particle for Infinite Particle Quantum Systems*

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Abstract. We give a simple proof that the ground state energy per particle for several interacting particle systems is monotone and bounded as the number of particles increases. Some of the systems for which the proof holds are anharmonic oscillator approximations to $|\phi|_d^4$ quantum fields, many body Schrödinger operators with nearest and next to nearest neighbor couplings, and systems whose energy is given by operators which are not restricted to being differential operators.

It is well known that if we pinch a vibrating system of particles, or clamp a vibrating membrane the fundamental frequencies increase. Similarly if we tear a vibrating membrane its fundamental frequency decreases. Some of the mathematical consequences of these observations, in classical physics, are that imposing Dirichlet or Neumann boundary conditions on $-\Delta$, (or finite difference approximations to $-\Delta$) along a curve in the interior of some bounded region, raises or lowers its eigenvalues [1].

Some of the mathematical consequences of these observations in quantum physics are described in this paper. We show in particular that the ground state energy per particle, for the sequence of coupled anharmonic oscillator Schrödinger operators with Dirichlet boundary conditions used in the "doubling algorithm" in [2] to numerically approximate the ϕ_2^4 quantum field theory, is monotone decreasing.

We next show that the ground state energy per particle, for the sequence of coupled anharmonic oscillator Schrödinger operators with Neumann Boundary conditions used in [2] to approximate the ϕ_2^4 theory, is monotone increasing.

We also show that the energy per particle is bounded from above and below in both of the preceding cases and hence it converges to a limit as the number of particles becomes infinite.

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