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The Borel Transform in Euclidean φ_ν^4 Local Existence for Re$\nu < 4$

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Abstract. We consider the φ^4 theory in Euclidean space of complex dimension ν and prove that, for Re$\nu<4$ the renormalized Feynman amplitudes grow at worst exponentially in the number of vertices in the graph. This implies that the Borel transform of any Schwinger function may be defined in a neighborhood of the origin in the Borel plane.

1. In this paper we prove the existence, in a neighborhood of the origin, of the Borel transform of the perturbative series for any Schwinger function of the Euclidean φ_ν^4 model, when the (possibly complex) dimension ν of space time satisfies Re$\nu<4$.

We do not discuss the more difficult problem of extending the domain of analyticity of the transform and proving the Borel summability of the theory (a problem which has been solved by constructive quantum field theory in the integer dimensions $\nu=1,2,3$). A discussion of the background and motivation for this study is given in [1] (see also [2]), to which we refer the interested reader; here we summarize the problem briefly.

We consider then a truncated Schwinger function of $N \geqq 2$ fields. Such a function has a formal power series development in the coupling constant λ:

$$\delta\left(\sum_1^N p_i\right) \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} a_n(p, \nu),$$

where a_n is a sum of Feynman amplitudes which are associated with connected Feynman graphs and in which ν appears as a parameter. Since the number of such graphs is obviously smaller than the total number of graphs appearing in the development of the full Schwinger function, and since this number $K_{N,n}=(4n+N-1)!!$ is certainly bounded by $(n!)^2 \cdot (16)^n\ (4n+N-1)^{N/2}$, the con-

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