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Duality and Absence of Locally Generated Superselection Sectors for CCR-Type Algebras

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Abstract. We isolate an abstract algebraic property which implies duality in all locally normal, irreducible representations of a quasilocal C^*-algebra if it holds together with two more specific conditions. All these conditions holding for the CCR-algebra in $d \geqq 2$ space time dimensions duality follows for representations of the two-dimensional CCR-algebra generated by pure Wightman states of $P(\Phi)_2$-theories. We then show that algebras of this kind have no nontrivial locally generated superselection sectors which for $d \geqq 3$ yields a first approximation to a quantum analogue of Derrick's theorem.

Preliminaries

Viewing the central role which duality plays in the abstract theory of superselection sectors [1] it is unfortunate that so far it has not been possible to prove this property in any nontrivial case (see, however, [2] where a suitable C^*-algebra is constructed from the given one—this will, however, hardly be representation independent). On account of this the present paper is good news for it remedies this situation. On the other hand the very property which makes this possible also implies that sectors generated by local automorphisms from a vacuum sector are unitarily equivalent to it—therefore in these cases nontrivial superselection sectors in the sense of [1] do not exist. Thus the present paper raises as many questions as it solves.

We let $\mathfrak{A}$ be our quasilocal C^*-algebra with its net of local von Neumann algebras $\{\mathscr{R}(\mathcal{O})\}$. To simplify matters we shall assume $\mathscr{R}(\mathcal{O})$ to be a factor if $\mathcal{O}$ is a bounded double cone (b.d.c.). For any b.d.c. $\mathcal{O}$ we denote by $\mathcal{O}^x$ the open interior of its causal complement and by $\mathfrak{A}(\mathcal{O}^x)$ the C^*-subalgebra of $\mathfrak{A}$ generated by all $\mathscr{R}(\tilde{\mathcal{O}})$ where $\tilde{\mathcal{O}}$ runs through all b.d.c.'s with $\tilde{\mathcal{O}} \ll \mathcal{O}^x$, i.e. $\bar{\tilde{\mathcal{O}}} \subset \mathcal{O}^x$. This definition is a matter of convenience and must be modified if there are observables in $\mathcal{O}^x$ which cannot be approximated by observables localized in b.d.c.'s contained in

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