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Completely Positive Quasi-Free Maps on the CAR Algebra

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Abstract. We construct and study a new class of quasi-free completely positive maps on the C^*-algebra of the canonical anti-commutation relations.

Introduction

We construct and study quasi-free completely positive maps between algebras of the canonical anti-commutation relations (CAR) induced by contractions at the underlying hilbert space level. We are motivated in part by some analogous results for the Boson case, which however were obtained with less effort using generating functions for example [7, 8, 13, 14, 16, 19, 20]. We take as our starting point for the CAR algebra, the theory of quasi-free states, quasi-free automorphisms and the work in [23, 31] (see also [9, 20]) on quasi-free completely positive maps which leave the Fock or anti-Fock state invariant.

In the first section, we consider a completely positive contraction on a C^*-algebra which possesses an invariant state. We study the given map with the aid of the contraction it induces in the GNS Hilbert-space of the invariant state, and show under suitable conditions that a completely positive contraction dominates its spatial part in the GNS decomposition of an invariant KMS state.

In Sect. 2 the theory of quasi-free states on the CAR algebra is reviewed and our notation established. In the third section we construct and develop the theory of a single completely positive quasi-free map. In particular, the main result of Sect. 1 is used to easily identify the pure ones.

In the final section we analyse semigroups of completely positive quasi-free maps on the CAR algebra. In particular, we show that the infinitesimal generator of such a semigroup is bounded at the C^*-level if and only if it is of trace class at the hilbert space level. This improves the results of [1] (see also [28]) for quasi-free derivations, and a partial result of [9] for quasi-free generators of Fock-type. Some dilation and perturbation questions are also discussed.

We work throughout with CAR algebras built over complex hilbert spaces, but much of our work also holds for those over real spaces.