On Positivity of Mass for Black Hole Space-Times

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Abstract. For any time-symmetric asymptotically flat initial slice with an apparent horizon, the associated four-momentum measured at spatial infinity is shown to be future-directed and time-like.

I. Introduction

Associated with any asymptotically flat space-time at spatial infinity there exists the notion of the total four-momentum, which is called Arnowitt-Deser-Misner (ADM) four-momentum [1–3]. If the asymptotically flat space-time is regular initially and matter fields are physically reasonable, one expects the ADM four-momentum to be future-directed and time-like. This idea goes under the name of the positive mass conjecture (at spatial infinity) [4].

Recently, important progress has been made in this problem by Schoen and Yau [5], who showed that the ADM four-momentum is future-directed with respect to a regular maximal slice whose positive-definite metric satisfies a certain asymptotic condition. However, their proof fails to resolve the full conjecture because of two somewhat related reasons: First, as York [6] pointed out, their asymptotic condition on the positive-definite metric of the slice is too restrictive from the physical point of view. For the Schwarzschild solution, although their condition is satisfied on a $t = \text{const}$ surface, it is not satisfied on a boosted asymptotically flat slice. Second, note that even though the existence of a regular initial slice is assumed in the positive mass conjecture, singularities are allowed to develop in the future of the slice. This could cause additional difficulties in establishing the existence of a regular maximal slice. Even if one could overcome these difficulties, the proof of the conjecture requires one to show that the four-momentum is time-like as well as future-directed. Only when the four-momentum has no spatial component with respect to a maximal slice, (e.g., when the slice is time-symmetric) is the positivity of energy, i.e., positivity of the time component of four-momentum, sufficient for the proof of the conjecture.

The full conjecture, thus, being still open, we would like to present another line of proof for a time-symmetric slice. In contrrast to Schoen and Yau's, our proof