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O(2) Symmetric Connections in an SU(2) Yang-Mills Theory**

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Abstract. The general O(2) symmetric Yang-Mills equations are derived. An ansatz for O(2) symmetric merons is presented and it is shown that any connection in this ansatz will have SU(2) topological charge density which is a sum of delta functions at points in a plane with weights $\pm\frac{1}{2}$. It is shown that any connection in this ansatz will be C^∞ away from these points.

1. Introduction

If the four-dimensional, Euclidean space, Yang-Mills equations are required to be O(3) symmetric, it was shown by Witten [1] that the equations reduced to the equations describing an interacting system on the Poincaré half-plane consisting of a $U(1)$ gauge field and a charged scalar field with self-couplings. DeAlfaro, Fubini, and Furlan [2] discovered an explicit O(3) symmetric solution to the Yang-Mills equations with the property that the topological charge density has values $\pm\frac{1}{2}$ concentrated at points; the 'two meron solution.' For arbitrary positive integer N, Glimm and Jaffe [3, 4] reduced the problem of finding N-meron, O(3) symmetric, solutions to the question of whether certain (infinite action) solutions to the scalar elliptic equation.

$$r^2(\partial_r^2 + \partial_r^2)\psi = \psi(\psi^2 - 1) \tag{1.1}$$

existed. These N-meron solutions of Glim and Jaffe are also characterized by a topological charge density which is equal to a sum of delta functions on a line with weights $\pm\frac{1}{2}$ at the merons. Jonsson, McBryan, Zirilli, and Hubbard [5] proved that these (infinite action) solutions to Eq. (1.1) do indeed exist. In this paper, an O(2) symmetric SU(2) Yang-Mills connection is defined and the form of the O(2) symmetric Yang-Mills equations are derived. It is then shown that certain classes of O(2) symmetric connections have topological charge densities which are the sum of delta functions in a plane with weights $\pm\frac{1}{2}$. The O(3) symmetric solutions of Glimm and Jaffe are included in these classes. It remains to be proven whether there are any other solutions to the Yang-Mills equations contained in these classes.

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