Commun. Math. Phys. 69, 101–129 (1979)

Communications in
Mathematical
Physics

The Quasi-Classical Limit of Quantum Scattering Theory

Kenji Yajima*

Mathematical Research Institute, ETH-Zürich, Switzerland, and Department of Mathematics, University of Tokyo, Tokyo, Japan**

Abstract. We study the quasi-classical limit of the quantum mechanical scattering operator for non-relativistic simple scattering system. The connection between the quantum mechanical and classical mechanical scattering theories is obtained by considering the asymptotic behavior as $\hbar \to 0$ of the quantum mechanical scattering operator on the state $\exp(-ip\cdot a/\hbar)f(p)$ in the momentum representation.

Introduction

Let us consider the Schrödinger operator

$$H^h = -\frac{\hbar^2}{2m}\Delta + V(x), \quad \Delta = \frac{\partial^2}{\partial x_1^2} + \dots + \frac{\partial^2}{\partial x_1^2} \tag{0.1}$$

in the Hilbert space $\mathscr{H} = L^2(\mathbb{R}^n)$ and let $H_0^h = -\frac{\hbar^2}{2m}\Delta$. Here $\hbar = \frac{h}{2\pi}$ and h is the small positive parameter called Planck's constant. We assume the potential $V(x)$ to satisfy the following condition.

Assumption (A). (1) $V(x)$ is a real valued infinitely differentiable function on $\mathbb{R}^n$.
(2) For any multi-index α, there exist constants $m(\alpha) > |\alpha| + 1$ and $C_\alpha > 0$ such that

$$\left|\left(\frac{\partial}{\partial x}\right)^\alpha V(x)\right| \leqq C_\alpha(1+|x|)^{-m(\alpha)}.$$

Under this condition H_0^h and H^h are self-adjoint operators with the domain $\mathscr{D}(H_0^h) = \mathscr{D}(H^h) = H^2(\mathbb{R}^n) =$ the Sobolev space of order 2. Furthermore it is well known that the wave operators $W_\pm^h$ defined as

$$W_\pm^h = \mathrm{s} - \lim_{t \to \pm\infty} e^{itH^h/\hbar} e^{-itH_0^h/\hbar}$$

exist and are complete:

---

* Partially supported by Fûjû-kai Foundation and Sakkô-kai Foundation

** Current address

0010-3616/79/0069/0101/$05.80