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Analyticity of the Pressure for Heisenberg and Plane Rotator Models

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Abstract. We use complex rotations and convex completion to extend the analyticity region of the pressure for the Heisenberg and plane rotator models.

1. Introduction

We consider partition functions for 2 or 3 component ferromagnets in a complex external field $(\mathbf{H}_j)_{j=1,\ldots,N}$, where N is the number of sites (eventually $N\to\infty$). Previous results [1,4] on the location of zeros are unsatisfactory, either because the region which is free of zeros has only N complex dimensions (instead of $2N$ or $3N$ according to the number of components), or because the real points inside this region are too restricted (external fields in a quadrant instead of a half space). An exception is Fröhlich's result [3] which however relies on the analyticity for large external fields: extra (but standard) assumptions are required, and the resulting region is a very small neighborhood of the real points.

In the first part of this paper, we consider rotation invariant models and extend the analyticity region (for the pressure) from N to $3N$ complex dimensions by the use of complex rotations (as in axiomatic field theory). If $\tilde{\mathbf{H}}_j=(H_j^z, iH_j^x, iH_j^y)$, the resulting analyticity region contains the forward tube $\{\mathrm{Re}\tilde{\mathbf{H}}_j\in V_+ : j=1,\ldots,N\}$ together with all its transforms under the real rotations of the original variables.

In the second part, we consider the anisotropic plane rotator model, for which both unsatisfactory results mentioned above are available. Analytic completion again gives analyticity in the forward tube.

2. Complex Rotation of the Heisenberg Model

We formulate the result for the quantum spin$\frac{1}{2}$ Heisenberg model which is more basic: it has a natural Ising spin approximation, and it has many descendents: arbitrary quantum spin, 3 and 2 component classical spins (rotators).

Theorem 1. *For $j=1,\ldots,N$, let S_j^z, S_j^x, S_j^y denote Pauli matrices acting on the j'th factor of the tensor product $\bigotimes_{j=1}^{N}\mathbb{C}^2$, and let $\mathbf{H}_j=(H_j^z, H_j^x, H_j^y)\in\mathbb{C}^3$. Given positive*

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