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A Note on the Vacuum Structure of an SU(2) Yang-Mills Theory

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Abstract. We discuss different compactifications of the spacial part $\mathbb{R}^3$ of Minkowski space and give classifications of the vacuum structure for a Yang-Mills theory.

1. Introduction

The possible vacua in an SU(2) Yang-Mills theory and their physical implications have been discussed in several papers [1]–[6]. It turns out that a classification of this vacuum structure can be given by using homotopy theory of the underlying topological spaces. If we restrict ourselves to the so called $A^0=0$ gauge, the vacuum configurations are given by pure gauge fields $\mathbf{A}(\mathbf{x})=g^{-1}(\mathbf{x})\nabla g(\mathbf{x})$, where $g:\mathbb{R}^3\to \mathrm{SU}(2)$ is some mapping of the spacial part $\mathbb{R}^3$ of Minkowski space into the gauge group G, which for simplicity we take to be SU(2). To get the above mentioned classification one proceeds as follows: one compactifies $\mathbb{R}^3$ to some compact space K and studies then the continuous mappings of K into the gauge group G. Commonly one takes for K the one-point compactification S^3 and gets then a vacuum classification, for instance, via $\pi_3(\mathrm{SU}(2))$ which is isomorphic to $\mathbb{Z}$. Therefore an infinite sequence $\mathbf{A}_n, n=0,\pm 1,\ldots,$ of vacua arises in an SU(2) theory.

In terms of the mappings $g:\mathbb{R}^3\to \mathrm{SU}(2)$ the one-point compactification can be described also by allowing only those mappings g which have the property that $\lim_{\mathbf{x}\to\infty} g(\mathbf{x})=\text{const.}$ independent of the direction in which one goes to infinity.

Now Gribov [5] found that the physical properties of such a theory are in a great deal affected also by mappings g which have a more complicated behaviour at infinity. It is therefore natural to look for different compactifications of $\mathbb{R}^3$ which allow also such mappings.

Instead of giving a compactification of the space $\mathbb{R}^3$ in terms of a certain topological compact space K in which it can be embedded we use another comple-

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