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Circuit Based Graphs for Renormalized Perturbation Theory*

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Abstract. A generalization of graph theory is introduced and used to obtain Feynman parametric formulas relevant to renormalized amplitudes. The generalization of graph theory is based upon circuit coefficients instead of the usual incidence matrix. The parametric formulas presented are valid for amplitudes which have been renormalized, as in the Zimmermann formulation, by subtracting Taylor terms in momentum space.

0. Introduction

In their beautiful work on graph theory and Feynman amplitudes, Nakanishi [1] and Speer [2] were able to analyze the ultraviolet singularities of the unsubtracted Feynman amplitude by applying graph theory to obtain formulas for the integrand in a Feynman parametric space.

For each circuit, C, of the graph representing the amplitude, they assign circuit coefficients (set indicator) according to the rule

$$\begin{aligned}(C;\ell) &= 1, \text{ if } \ell \text{ is a line of } C, \text{ and}\\ &= 0, \text{ otherwise.}\end{aligned}$$

These coefficients may be identified to within a sign with the numbers d_ℓ^i for which

$$p_\ell = \sum_{i=1}^{m} d_\ell^i k_i$$

gives the internal momentum of the ℓ-th line of the graph. We will call the d_ℓ^i momentum routing coefficients.

In the Zimmermann [3] formulation of the BPH [4, 5] subtraction procedure, renormalization is accomplished by subtracting certain Taylor terms in

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