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Preturbulence: A Regime Observed in a Fluid Flow Model of Lorenz*

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Abstract. *This paper studies a forced, dissipative system of three ordinary* differential equations. The behavior of this system, first studied by Lorenz, has been interpreted as providing a mathematical mechanism for understanding turbulence. It is demonstrated that prior to the onset of chaotic behavior there exists a preturbulent state where turbulent orbits exist but represent a set of measure zero of initial conditions. The methodology of the paper is to postulate the short term behavior of the system, as observed numerically, to establish rigorously the behavior of particular orbits for all future time. Chaotic behavior first occurs when a parameter exceeds some critical value which is the first value for which the system possesses a homoclinic orbit. The arguments are similar to Smale's "horseshoe".

Section 1. Introduction and Definitions

Many mathematical attempts have been made to interpret the phenomenon of turbulence in fluids. Typically, the behavior of the fluid is represented by the trajectories of a system of differential equations and the system is assumed to depend on a parameter r. The parameter r usually corresponds to the Rayleigh or Reynold's number. [For a more detailed discussion of alternate interpretations of turbulence refer to [1].]

One of the most intriguing models of this type was studied by Lorenz [2]. Lorenz considered the forced dissipative system

$$\begin{aligned} x' &= -\sigma x + \sigma y \\ y' &= -xz + rx - y, \\ z' &= xy - bz. \end{aligned} \tag{1.1}$$

These ordinary differential equations are an approximation to a system of partial differential equations describing finite amplitude convection in a fluid layer heated

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