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Decay of a Yang-Mills Field Coupled to a Scalar Field*

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Abstract. Consider a gauge field F and a scalar field ϕ with a self-coupling $V(\phi)$ as well as the standard coupling between F and ϕ. If $0 \leqq 2V(\phi) \leqq \phi \cdot V'(\phi)$, there are no classical lumps. If $V(\phi) = |\phi|^4$ the system is conformally invariant and all the energy radiates out along the light cone.

1. Introduction

Consider the Lagrangian density

$$\mathscr{L} = -\tfrac{1}{4}F^2 + \tfrac{1}{2}(D\phi)^2 - V(\phi) \tag{1}$$

in Minkowski space, where F denotes a Yang-Mills field, ϕ a scalar field, and V a self-coupling depending only on $|\phi|^2$. The internal symmetry group $\mathfrak{G}$ is a compact Lie group, and D denotes the covariant derivative.

In [1] we considered a pure Yang-Mills field with Lagrangian F^2. We proved that all the energy of a solution to the Yang-Mills equations radiates out along the light cone. We prove here the exact analogue in the case of (1) provided V satisfies the inequality

$$0 \leqq 4V(\phi) \leqq \phi \cdot V'(\phi). \tag{2}$$

A typical case is $V(\phi) = c|\phi|^p$ where $c > 0$, $p \geqq 4$.

In case $V(\phi) = c|\phi|^4$, our Lagrangian $\mathscr{L}$ is invariant under the conformal group of Minkowski space (Theorem 1). This 15 dimensional Lie group generates 15 conservation laws satisfied by the solutions of the equations of motion. We derive the explicit forms of these laws in Sect. 3. In Sect. 4 we show that one of them, the First Inversional Law, implies the decay result mentioned above. For arbitrary V, they are no longer all conservation laws. However, under assumption (2) the extra terms in the First Inversional Law have the proper signs and we infer the same decay result.

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