

One-Dimensional DLR Invariant Measures are Regular

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Abstract. A system of infinite spins in one dimension is considered. The interaction is given by a pair potential $-J_{xy}S_xS_y$, where S_x, S_y are the spins at the sites $x, y \in \mathbb{Z}$ and $J_{xy} = J(|x - y|)$ where $J(|x - y|)$ decreases asymptotically in an integrable way. The self-interaction makes the system superstable. It is proven that any invariant DLR measure for this system satisfies Ruelle's superstable estimates (regularity condition).

1. Introduction

Gibbs random fields (DLR measures) with random variables taking on values in non-compact spaces have been extensively studied, [1–5] for applications both in statistical mechanics and lattice approximation to constructive field theory (point particles without hard-core interaction, infinite spin systems). Under reasonable hypotheses on the interaction it has been proven that there exists a set of so called regular DLR measures [see Eq. (1.2) below and Sect. 2] which have essentially the same properties as the fields in the compact case, [1–4]. The regular measures are characterized by support properties and can be explicitly constructed by taking thermodynamic limits of finite volume Gibbs measures with boundary conditions taken within an appropriate class, [3, 4]. One is therefore induced to consider these as the physically interesting states; it is known, however, that, in general, one cannot exclude the presence of other non regular DLR measures. These have been explicitly constructed for a chain of one-dimensional nearest neighborhood interacting harmonic oscillators, [6]. To obtain them one starts with finite volume Gibbs measures and takes boundary conditions which are fastly increasing as the volume gets to infinity. The limiting measures keep memory of this process and the mean values of the harmonic oscillators are very large and not uniformly bounded. Therefore one needs “very large boundary conditions” for the limiting measure to be non regular, but this automatically leads toward a non homogenous situation. From this follows the conjecture: the translationally invariant DLR measures are regular (which is the case in the above mentioned, harmonic oscillators case).