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The Partition Function of a Degenerate Functional

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Abstract. The partition function of a degenerate quadratic functional is defined and studied. It is shown that Ray-Singer invariants can be interpreted as partition functions of quadratic functionals. In the case of a degenerate non-quadratic functional the semiclassical approximation to the partition function is considered.

Section 1. Introduction

The degenerate Lagrangians are important in quantum field theory. For example the action in gauge theories is invariant with respect to infinite-dimensional group of local gauge transformations and therefore the corresponding Lagrangian is degenerate. To calculate the physical quantities in gauge theories one must impose the gauge conditions, but final results must be independent of the gauge conditions. The physical quantities in the gauge theories and other theories described by degenerate Lagrangians were expressed through functional integrals by Faddeev and Popov (see [1]).

In the present paper we give a rigorous treatment of the case when the action is a degenerate quadratic functional (Sects. 2 and 3). Our results can be useful when dealing with various questions on quantum field theory. For example, they are connected with so-called anomalies. These results can be used to study the instanton contribution in Schwinger functions (Sect. 5). Our assertions can be applied outside of quantum field theory too. They are closely related with the theorems proved in [2, 3]. Namely, we show that the Ray-Singer torsion [2] can be considered as a partition function of action which is invariant by diffeomorphisms. The independence of Ray-Singer torsion on the choice of riemannian metric can be interpreted as independence of the partition function on the choice of gauge condition. In a similar way one can get invariants constructed in [37] and new invariants. One of the new invariants will be described below.

Part of our results was formulated in [4]. A short review of some mathematical results used in present paper can be found in [5].

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