

Born Series for (2 Cluster) \rightarrow (2 Cluster) Scattering of Two, Three, and Four Particle Schrödinger Operators

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Abstract. We investigate elastic and inelastic (2 cluster) \rightarrow (2 cluster) scattering for classes of two, three, and four body Schrödinger operators $H = H_0 + \sum_{i < j} V_{ij}$.

Formulas are derived for those generalized eigenfunctions of H which correspond asymptotically in the past to two freely moving clusters. With these eigenfunctions, we establish a formula for the (2 cluster) \rightarrow (2 cluster) T -matrix and prove the convergence of a Born series for the T -matrix at high energy.

1. Introduction

In this paper we investigate (2 cluster) \rightarrow (2 cluster) scattering of certain classes of two, three, and four particle Schrödinger operators. We begin by finding those generalized eigenfunctions of the Hamiltonian, $\phi_\alpha(X, k)$, which correspond to two cluster initial channels α . Using these eigenfunctions we prove the validity of a formula for the physicists' T -matrix for (2 cluster) \rightarrow (2 cluster) elastic and inelastic processes. We then prove the convergence of a Born series expansion for the T -matrix at high energy.

In the two body case, we have little to say which is new. Eigenfunction expansions for two body Hamiltonians are developed in [2, 11, 14, 17, 20]. The two body T -matrix formula is proved in [17, 20]. Also, for each potential V in certain classes, there exists $E_0 < \infty$, such that the Born series converges for energies in (E_0, ∞) [1, 6, 15, 17, 20, 24]. However, our methods and the closely related methods of [15] are presently the only methods which can be used to estimate E_0 .

Our principal new results deal with 3 and 4 body systems. Previous authors [5, 13, 19, and references therein] have obtained the asymptotic behavior and distributional Fourier transforms of the generalized eigenfunctions. They have not proved Born series convergence nor the validity of the T -matrix formula [Eq. (1.2)].

* Supported in part by the National Science Foundation under Grant PHY 78-08066